GPO Reading 6 16.1.20

Sherali-Adams Hierarchie

Situation:

$P = P^{\geq}(A,b) \subseteq [0,1]^n$

Ziel: Beschränke $P_I = \mathrm{conv}(P \cap \mathbb{Z}^n)$
$(= \mathrm{conv}(P \cap \{0,1\}^n))$

möglichst genau

Beispiele/Erinnerung:

① Matching

$G = (V, E)$

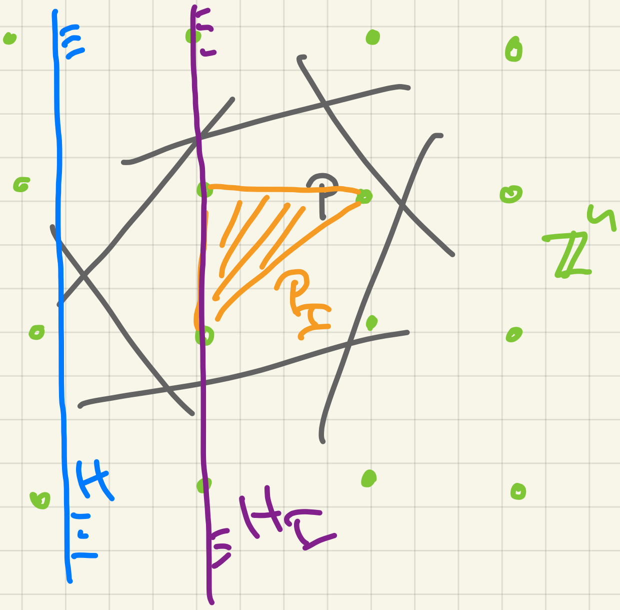
$P = \{x \in [0,1]^E : x(\delta(v)) \leq 1 \ \forall v \in V\}$

$P \cap \mathbb{Z}^n = \{X(M) : M \subseteq E \text{ Matching}\}$

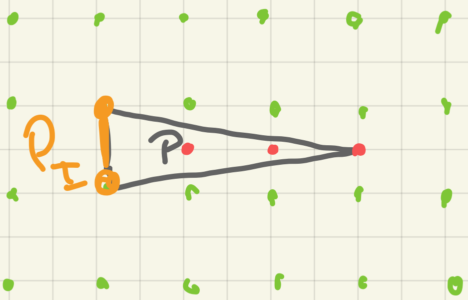
VL GanzOpt: $P_I \subseteq P' \subseteq P$

hier sogar "="

$$P' = \bigcap_{\substack{H \text{ Halbraum} \\ P \subseteq H}} H_I$$



I.A.: $P' \neq P_I$



② Stable Sets

$$G = (V, E)$$

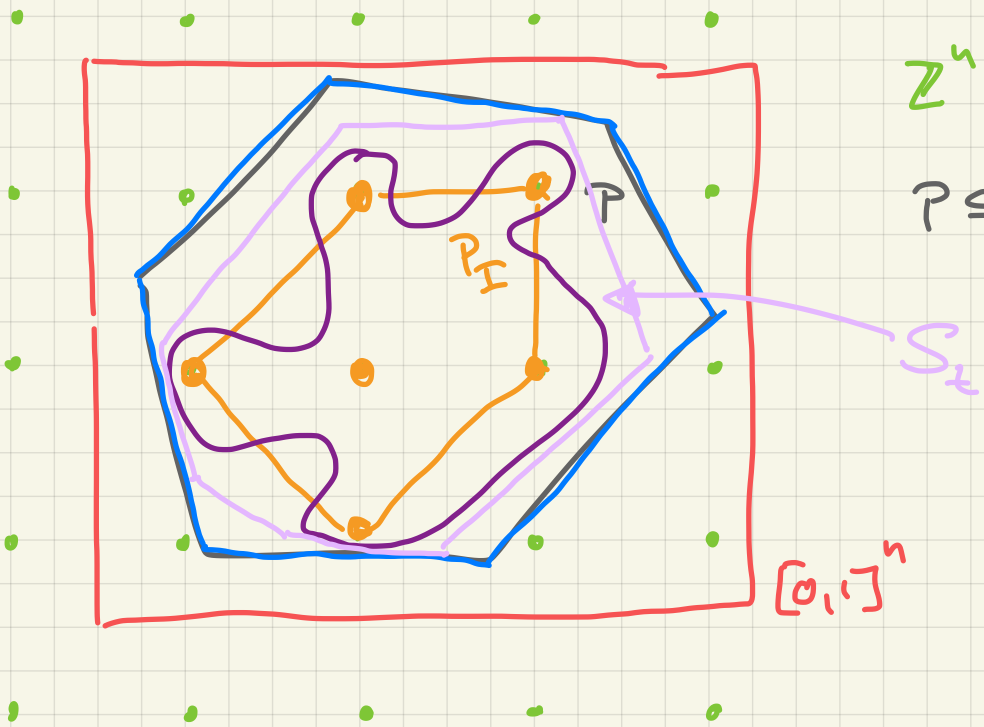
$$P = \{x \in [0,1]^V : x_v + x_w \le 1 \ \forall vw \in E\}$$

$$P \cap \mathbb{Z}^V = \{\chi(S) : \underbrace{S \subseteq V \text{ stabil}}_{\text{d.h. } \forall v,w \in S\,:\ vw \notin E}\}$$

$$P_I \underset{(\text{i.A.})}{\subsetneq} P' \underset{(\text{i.A.})}{\subsetneq} P$$

Sherali-Adams

(schematisch)



$$\left\{ x \in \mathbb{R}^n : (Ax - b) \cdot \prod_{i \in I} x_i \cdot \prod_{j \in J} (1 - x_j) \geq 0 \quad \forall\, 0 \leq |I| + |J| \leq t,\ I \cap J = \emptyset \right\}$$

$$\left\{ x \in \mathbb{R}^n : \text{———————} \ \text{"} \ \text{———————}, \quad x_i^2 = x_i \quad \forall i \in [n] \right\}$$

$$y_I := \prod_{i \in I} x_i \qquad \forall\, 0 \leq |I| \leq t+1$$

$$\rightsquigarrow \quad R_t \subseteq \mathbb{R}^{\mathfrak{P}_n} \qquad \left(\mathfrak{P}_n = 2^{[n]} \right)$$

$S_t :=$ Projektion von R_t auf x-Variablen $(x_i = y_{\{i\}})$

Ungleichungen $\sim \sum_{\tau=0}^{t} \binom{n}{\tau} \cdot 2^{\tau}$ ← # Möglichkeiten Auftrag in I und J

↑ # Möglichkeiten von $|I \cup J| = \tau$

Für $t \leq \frac{n}{2}$: $\sim \binom{n}{t} \cdot 2^{t} \sim n^{t}$

Es gilt: $P_I \subseteq \dots \subseteq S_{t+1} \subseteq S_t \subseteq \dots \subseteq P$

$[R_{t+1} \subseteq R_t]$